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Investigation of $g_{f_0\rho\gamma}$ and coupling constants in light cone sum rules

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Abstract. We present a calculation of the coupling constant of $f_0 \to \rho\gamma$ and $a_0 \to \rho\gamma$ decays from the point of view of the light cone QCD sum rules. We estimate the coupling constants $g_{f_0\rho\gamma}$, which are an essential ingredient in the analysis of physical processes involving the isoscalar $f_0(980)$ and the isovector $a_0(980)$ mesons.

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1 Introduction

To determine the fundamental parameters of hadrons from experiment, some information about physics at large distances is required. As is known, at large distance physics calculations cannot be done directly from the fundamental QCD Lagrangian, because at large distance perturbation theory cannot be applied. For this reason a reliable nonperturbative approach is needed. The QCD sum rules [1] have proved to be very useful to extract the low-lying hadron masses and coupling constants. This method is a framework which connects hadronic parameters with QCD parameters. It is based on a short-distance operator product expansion (OPE) in the deep Euclidean region of the vacuum–vacuum correlation function in terms of quark and gluon condensates. Further progress has been achieved by an alternative method known as the QCD sum rule method on the light cone [2–5]. The light cone QCD sum rule method has also been used to analyze several hadronic properties and to calculate hadronic coupling constants. The method of light cone sum rules is a hybrid of the standard technique of QCD sum rules in the manner of Shifman, Vainshtein, and Zakharov (SVZ) [1] with the conventional distribution amplitude description of the hard exclusive process [6, 7]. The basic idea of the SVZ sum rules is to use vacuum condensates to parameterize the non-trivial QCD vacuum and employ the duality hypothesis to relate the experimental observables to the theoretical calculation. Technically, an operator product expansion based on the canonical dimension is used. The difference between SVZ sum rules and light cone sum rules is that the short-distance Wilson OPE in increasing dimension is replaced by a light cone expansion in terms of distribution amplitudes of increasing twist.

With increasing experimental information about the different members of the meson spectrum it becomes very important to develop a consistent understanding of the observed mesons from a theoretical point of view. For the low-lying pseudoscalar, vector, and tensor mesons this has been done quite successfully within the framework of the simple quark model assuming the mesons to be quark–antiquark ($q\bar{q}$) states grouped together into nonets. For the scalar mesons, however, several questions still remain to be answered, most of them related to the nature of the experimentally observed mesons $f_0(980)$ and $a_0(980)$.

The light mesons have been the subject of continuous interest in hadron spectroscopy. Although the structure of these light mesons have not been unambiguously determined yet [8–10], the possibility may be suggested that these nine scalar mesons below 1 GeV may form a scalar $SU(3)$ flavor nonet [11]. In the naive quark model, we have $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ and $f_0 = s\bar{s}$, while in the framework of four-quark models, the mesons $f_0(980)$ and $a_0(980)$ could either be compact objects, i.e. nucleon-like bound states of quarks with the symbolic quark structures $f_0 = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$ and $a_0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$ [12–15], or spatially extended objects, i.e. deuteron-like bound states of hadrons; the $f_0(980)$ and $a_0(980)$ mesons are usually taken as KK molecules [9, 16, 17]. The nature of the meson $f_0(980)$ is particularly debated. One of the oldest suggestions is the proposal that quark confinement could be explained through the existence of a state with vacuum quantum numbers and mass close to the proton mass [18]. While considering the strong coupling to kaons, $f_0(980)$ could be interpreted as an $s\bar{s}$ state [19–23]. On the other hand, it does not explain the mass degeneracy between $f_0(980)$ and $a_0(980)$ interpreted as a $(u\bar{u} \pm d\bar{d})/\sqrt{2}$ state. A four-quark $qq\bar{q}\bar{q}$ state definition has also been offered [12, 13]. The identification of the f_0 and of the other lightest scalar

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mesons with the Higgs nonet of a hidden $U(3)$ symmetry has also been proposed [24]. On the other hand, they are relevant hadronic degrees of freedom, and therefore the role they play in hadronic processes should also be studied beside the questions of their nature. In this work, we calculated the coupling constant $g_{f_0\rho\gamma}$ and $g_{a_0\rho\gamma}$ by applying the light cone sum rule, which provides an efficient and model-independent method to study many hadronic observables, such as decay constants and form factors, in terms of non-perturbative contributions proportional to the quark and gluon condensates [5]. On the other hand we are also calculating the same coupling constants in the framework of QCD sum rules.

2 The light cone sum rules calculation

We choose the interpolating current for the ρ and $f_0(a_0)$ mesons as $j_\mu^{\rho 0} = \frac{1}{2}\left(\bar{u}\gamma_\mu^a u^a - \bar{d}\gamma_\mu^a d^a\right)$, $J_{f_0} = \frac{1}{\sqrt{2}}\left(\bar{u}^b u^b + \bar{d}^b d^b\right) \sin\theta + \cos\theta_s \bar{s}$, and $J_{a_0} = \frac{1}{2}\left(\bar{u}^b u^b - \bar{d}^b d^b\right)$ [25] respectively. The ρ-meson consists of u- and d-quarks, and the s-quark does not contribute in this calculation. $J_\nu^\gamma = e_u(\overline{u}\gamma_\nu u) + e_d(\overline{d}\gamma_\nu d)$ is the electromagnetic current with e_u and e_d being the quark charges.

In order to derive the light cone QCD sum rule for the coupling constants $g_{f_0\rho\gamma}$, we consider the following two point correlation function:

$$T_\mu(p,p') = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}p'\cdot x} \left\langle 0 \,|\, T\{j_\mu^\rho(x) j_s(0)\} \,|0\right\rangle_\gamma , \quad (1)$$

where γ denotes the external electromagnetic field, and j_μ^ρ and j_s and are the interpolating currents for the ρ meson and $f_0(a_0)$, respectively.

We therefore saturate the dispersion relation satisfied by the vertex function T_μ by these lowest-lying meson states in the vector and the scalar channels, and in this way we obtain for the physical part at the phenomenological level that (1) can be expressed by

$$T_\mu(p,p') = \frac{\langle 0 \,|\, j_\nu^\rho \,|\rho\rangle \,\langle\rho(p') \,|\, S(p)\rangle_\gamma \,\langle S \,|\, j_s \,|0\rangle}{(p'^2 - m_\rho^2)(p^2 - m_s^2)} . \quad (2)$$

In this calculation the full light quark propagator with both perturbative and non-perturbative contribution is used, and it is given as [26]

$$\begin{aligned} \mathrm{i}\,\mathrm{Im}(x,0) &= \langle 0 \,|\, T\{\bar{q}(x) q(0)\} \,|0\rangle = \mathrm{i}\frac{\not{x}}{2\pi^2 x^4} - \frac{\langle\bar{q}q\rangle}{12} \\ &- \frac{x^2}{192} m_0^2 \langle\bar{q}q\rangle - \mathrm{i}g_s \frac{1}{16\pi^2} \\ &\times \int_0^1 \mathrm{d}u \left\{ \frac{\not{x}}{x^2}\sigma_{\mu\nu} G^{\mu\nu}(ux) - 4\mathrm{i}u \frac{x_\mu}{x^2} G^{\mu\nu}(ux)\gamma_\nu \right\} + \ldots , \end{aligned} \quad (3)$$

where the terms proportional to the light quark masses, m_u or m_d, are neglected. After a straightforward computation we have

$$\begin{aligned} T_\mu(p,q) = 2\mathrm{i} \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}px} A(x_\rho g_{\mu\tau} - x_\tau g_{\mu\rho}) \\ \times \langle\gamma(q) \,|\, \bar{q}(x)\sigma_{\tau\rho} q(0) \,|0\rangle , \end{aligned} \quad (4)$$

where $A = \frac{\mathrm{i}}{2\pi^2 x^4}$. In order to evaluate the two point correlation function further, we need the matrix elements $\langle\gamma(q) \,|\, \bar{q}(x)\sigma_{\tau\rho} q(0) \,|0\rangle$. This matrix element can be expanded in the light cone photon wave function [27, 28],

$$\begin{aligned} &\langle\gamma(q) \,|\, \bar{q}\sigma_{\alpha\beta} q \,|0\rangle = \mathrm{i}e_q \langle\bar{q}q\rangle \\ &\times \int_0^1 \mathrm{d}u \mathrm{e}^{\mathrm{i}uqx} \{ (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \left[\chi\varphi(u) + x^2 \left[g_1(u) - g_2(u)\right]\right] \\ &+ \left[qx\,(\varepsilon_\alpha x_\beta - \varepsilon_\beta x_\alpha) + \varepsilon x\,(x_\alpha q_\beta - x_\beta q_\alpha)\right] g_2(u) \} , \end{aligned} \quad (5)$$

where e_q is the corresponding quark charge, χ is the magnetic susceptibility, $\varphi(u)$ is of leading twist two, and $g_1(u)$ and $g_2(u)$ are the twist four photon wave functions. The main difference between the traditional QCD sum rules and the light cone QCD sum rules is the appearance of these wave functions. Light cone QCD sum rules correspond to summation of an infinite set of terms in the expansion of this matrix element on the traditional sum rules. The price one pays for this is the appearance of a priori unknown photon wave functions. After evaluating the Fourier transform for the M_1 structure and then performing the double Borel transformation with respect to the variables $Q_1^2 = -p^2$ and $Q_2^2 = -p'^2$, we finally obtain the following sum rule for the coupling constant $g_{f_0\rho\gamma}$:

$$\begin{aligned} g_{f_0\rho\gamma} = \frac{1}{\sqrt{2}} \frac{m_\rho\langle\bar{u}u\rangle}{\lambda_{f_0}\lambda_\rho} \mathrm{e}^{m_\rho^2/M_1^2} \mathrm{e}^{m_{f_0}^2/M_2^2} \\ \times \left\{-M^2\chi\phi(u_0) E_0(s_0/M^2) + 4g_1(u_0)\right\} \sin\theta , \end{aligned} \quad (6)$$

and for $g_{a_0\rho\gamma}$:

$$\begin{aligned} g_{a_0\rho\gamma} = \frac{1}{6} \frac{m_\rho\langle\bar{u}u\rangle}{\lambda_{a_0}\lambda_\rho} \mathrm{e}^{m_\rho^2/M_1^2} \mathrm{e}^{m_{a_0}^2/M_2^2} \\ \times \left\{-M^2\chi\phi(u_0) E_0(s_0/M^2) + 4g_1(u_0)\right\} , \end{aligned} \quad (7)$$

where the function

$$E_0(s_0/M^2) = 1 - \mathrm{e}^{-s_0/M^2} \quad (8)$$

is the factor used to subtract the continuum, s_0 being the continuum threshold, and

$$u_0 = \frac{M_2^2}{M_1^2 + M_2^2} , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} , \quad (9)$$

where M_1^2 and M_2^2 are the Borel parameters in the ρ and $f_0(a_0)$ channels.

3 Numerical calculation

In our calculations we used the numerical values $\langle \bar{u}u \rangle = -0.014\,\text{GeV}^3$, $m_{f_0} = 0.98\,\text{GeV}$, $m_{a_0} = 0.98\,\text{GeV}$, $\lambda_{f_0} = 0.18 \pm 0.02\,\text{GeV}^2$ [29], $\lambda_{a_0} = 0.21 \pm 0.05\,\text{GeV}^2$ [30], $m_\rho = 0.77\,\text{GeV}$. We note that neglecting the electron mass the e^+e^- decay width of the ω meson is given as $\Gamma(\rho \to e^+e^-) = \frac{4\pi\alpha^2}{3m_\rho^3}\lambda_\rho^2$. Then using the value from the experimental leptonic decay width of $\Gamma(\rho \to e^+e^-) = 6.85 \pm 0.11\,\text{keV}$ for ρ [29, 31], we obtain the value $\lambda_\rho = 0.118 \pm 0.001\,\text{GeV}^2$ for the overlap amplitude of the ρ meson. In order to examine the dependence of $g_{f_0\rho\gamma}$ and $g_{a_0\rho\gamma}$ on the Borel masses M_1^2 and M_2^2, we choose $M_1^2 = M_2^2 = M^2$. Since the Borel mass M^2 is an auxiliary parameter, and the physical quantities should not depend on it, one must look for the region where $g_{f_0\rho\gamma}(g_{a_0\rho\gamma})$ is practically independent of M^2. Firstly, we determined that this condition is satisfied in the interval $1.4\,\text{GeV}^2 \le M^2 \le 2.4\,\text{GeV}^2$ for $g_{f_0\rho\gamma}$. The variation of the coupling constant $g_{f_0\rho\gamma}$ as a function of the Borel parameter M^2 at different θ's and a constant value of the continuum threshold at $s_0 = 2.0$ are shown in Fig. 1. Examination of this figure shows that the sum rule is rather stable with these reasonable variations of M^2. In the $f_0 \to \rho\gamma$ decay, we then choose the middle value $M^2 = 1.9\,\text{GeV}^2$ for the Borel parameter in its interval of variation and obtain the coupling constant of $g_{f_0\rho\gamma}$ for various angles θ and where only the error arising from the numerical analysis of the sum rule is considered.

We also use the numerical values mentioned above for the magnetic susceptibility $\chi = 3.15\,\text{GeV}^{-2}$ [32]. Using the conformal invariance of QCD up to one loop order, the photon wave functions can be expanded in terms of Gegenbauer polynomials, each term corresponding to contributions from operators of various conformal spins. Due to conformal invariance of QCD up to one loop, each term in this expansion is renormalized separately and the form of these wave functions at a sufficiently high scale is well known. In previous work, twist four photon wave functions were used [27, 28]. Since they are not correct one should use the new functions that have been calculated by Ball, Braun and Kivel [32], and hence we have used the asymptotic forms of the photon wave function given by [32]

$$\varphi(u) = 6u(1-u)\,,$$
$$g_1(u) = -\frac{a}{16} - \frac{h}{8}\,, \tag{10}$$

where $a = 40u^2(1-u)^2$ and $h = -10(1-6u+6u^2)$ [33]. We will set $M_1^2 = M_2^2 = 2M^2$ which sets $u = u_0 = 1/2$. Note that in this approximation, we only need the value of the wave functions at a single point, namely at $u_0 = 1/2$ and hence the functional forms of the photon wave functions are not relevant.

In Fig. 2, we also showed the dependence of the coupling constant $g_{f_0\rho\gamma}$ on the parameter M^2 at some different values of the continuum threshold as $s_0 = 1.6$, 1.8 and $2.0\,\text{GeV}^2$ at $\theta = 30°$. Since the Borel masses M_1^2 and M_2^2 are the auxiliary parameters and the physical quantities should not depend on them, one must look for the region where $g_{f_0\rho\gamma}$ is practically independent of M_1^2 and M_2^2. We determined that this condition is satisfied in the interval $1.4\,\text{GeV}^2 \le M_1^2 \le 2.4\,\text{GeV}^2$. Choosing the middle value GeV^2 for the Borel parameter in this interval of variation we have the coupling constant of $g_{f_0\rho\gamma}$ for different s_0 values between $g_{f_0\rho\gamma} = 2.85 \pm 0.34$ and $g_{f_0\rho\gamma} = 2.69 \pm 0.32$ in the calculation of the light cone sum rules. In Fig. 3 is given the dependence of the coupling constant $g_{a_0\rho\gamma}$ on the parameter M^2 at some different values of the continuum threshold, $s_0 = 1.6$, 1.8 and $2.0\,\text{GeV}^2$. We have the interval of $1.4\,\text{GeV}^2 \le M_1^2 \le 2.0\,\text{GeV}^2$; then choosing the middle value $M^2 = 1.7\,\text{GeV}^2$ for the Borel parameter in this interval of variation we have the coupling constant of $g_{a_0\rho\gamma}$ for different s_0 values between $g_{a_0\rho\gamma} = 1.18 \pm 0.27$ and $g_{a_0\rho\gamma} = 1.22 \pm 0.28$. The coupling constant $g_{a_0\rho\gamma}$ was calculated [29] as 2.0 ± 0.50 and 1.30 ± 0.30 by QCD sum rules.

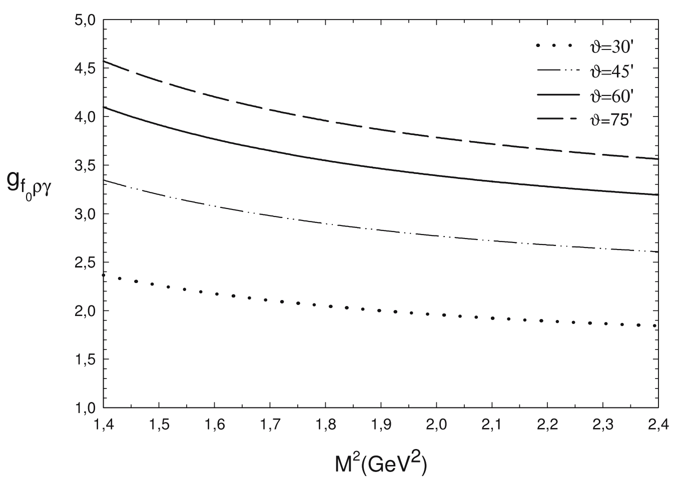

Fig. 1. The variation of the coupling constant $g_{f_0\rho\gamma}$ as a function of the Borel parameter M^2 at different θ and for a constant value of the continuum threshold at $s_0 = 2.0$

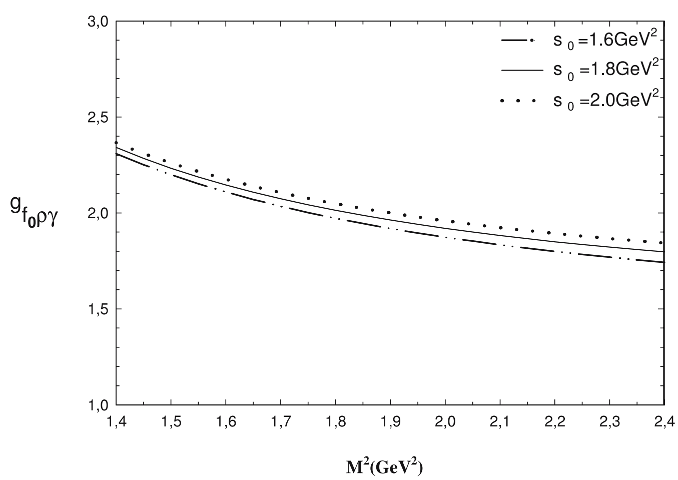

Fig. 2. The dependence of the coupling constant $g_{f_0\rho\gamma}$ on the parameter M^2 at some different values of the continuum threshold, $s_0 = 1.6$, 1.8 and 2.0 GeV at $\theta = 30°$

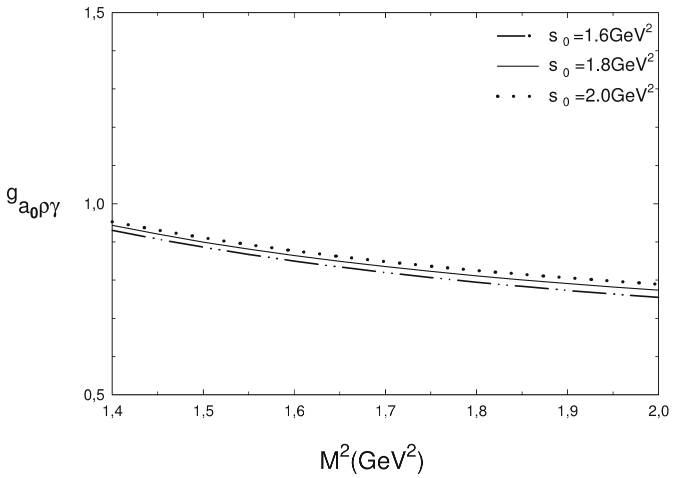

Fig. 3. The dependence of the coupling constant $g_{a_0\rho\gamma}$ on the parameter M^2 at some different values of the continuum threshold, $s_0 = 1.6$, 1.8 and 2.0 GeV^2

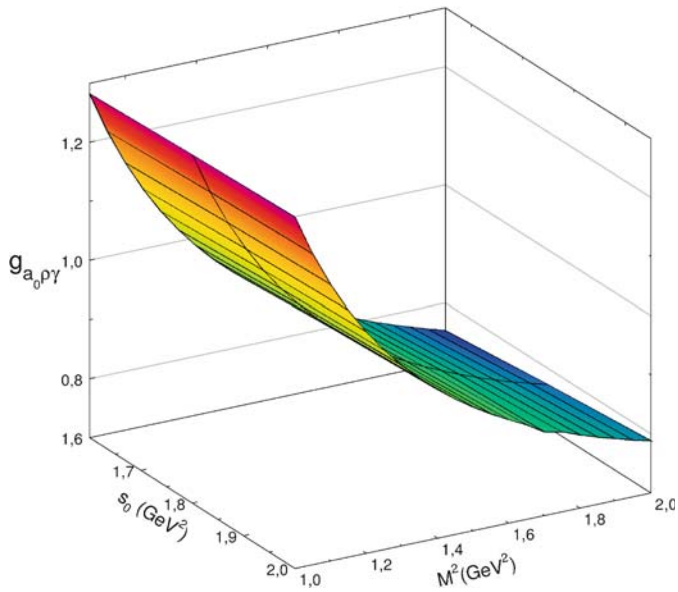

Fig. 5. Coupling constant $g_{a_0\rho\gamma}$ as a function of M^2 and s_0

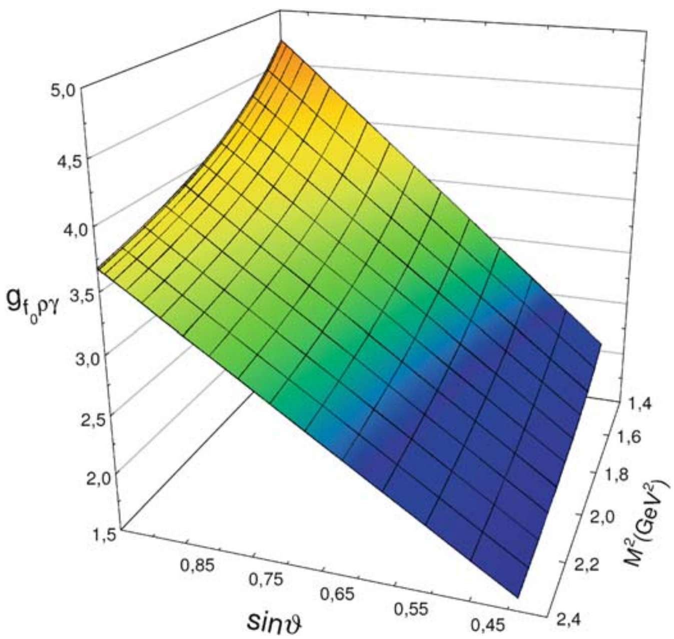

Fig. 4. Coupling constant $g_{f_0\omega\gamma}$ as a function of M^2 and $\sin\theta$

It follows from Figs. 2 and 3 that for the choices between $s_0 = 1.6, 1.8$ and 2.0 GeV^2 the variations in the result is about $\sim 10\%$, i.e., the coupling constant can be said to be essentially independent of the continuum threshold s_0. Furthermore, the coupling constant seems to be insensitive to variation of the Borel parameter M^2. Here all possible sources of uncertainties are taken into account, namely, errors coming from a determination of λ_{f_0}, from variation of the continuum threshold s_0, Borel parameters M^2, and errors in the values of the condensates. Along these lines, we calculated the coupling constant $g_{f_0\rho\gamma} = 2.85 \pm 0.57$ and $g_{a_0\rho\gamma} = 1.22 \pm 0.36$. The variation of the coupling constant $g_{f_0\rho\gamma}$ as a function of different values M^2 and θ are given in Fig. 4. Examination of this figure points out that the sum rule is rather stable with these reasonable variations of M^2. In Fig. 5 the relation between the different values of M^2 and s_0 is given for the variation of the coupling constant $g_{a_0\rho\gamma}$.

4 Conclusions

In this study we calculated the coupling constants $g_{f_0\rho\gamma}$ in the framework of light cone sum rules where the u- and d-quark contributions are considered. Generally, even the light cone sum rule works much better for heavy quarks but it can also give good results for light quarks in some calculations. We applied the LCSR method by using the most correct wave functions. In spite of lacking experimental data on $g_{f_0\rho\gamma}$, we found estimated values for the coupling constant $g_{f_0\rho\gamma}$ and $g_{a_0\rho\gamma}$ in the light cone sum rules. The results depend on the mixing angle θ and the s_0 parameter for $g_{f_0\rho\gamma}$. If we choose the current as $J_{f_0} = \frac{1}{2}\left(\bar{u}^b u^b + \bar{d}^b d^b\right)$, one then has to change (6) so in that case we get $g_{f_0\rho\gamma} = 2.76 \pm 0.79$.

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